MATHEMATICS

32

PAPER—II

Time Allowed : Three hours | *Maximum Marks* : 300

The figures in the margin indicate full marks for the questions

Candidates should answer Question Nos. **1** and **5** which are compulsory and *any* **three** from the rest selecting at least **one** from each Section

SECTION—A

1. Answer any *five* from the following : 12×5=60

(a) If H is a subgroup of G, let $N(H) = \{g \in G \mid gHg^{-1} = H\}$. Prove that

(i) $N(H)$ is a subgroup of G

(ii) H is normal in $N(H)$

(b) Let f and g be continuous on $[a, b]$. Prove that $f + g$ and $f \cdot g$ are also continuous on $[a, b]$.

(c) Prove that the complex valued function $f(z)$ defined by

$$f(z) = \begin{cases} \dfrac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}, & \text{if } z = x + iy \neq 0 \\ 0, & \text{if } z = 0 \end{cases}$$

is continuous and the Cauchy-Riemann equations are satisfied at the origin, yet $f'(0)$ does not exist.

(d) Using graphical method, solve the following LPP :

$$\text{Max } Z = 5x_1 + 3x_2$$

subject to

$$3x_1 + 5x_2 \leq 15$$
$$5x_1 + 2x_2 \leq 10$$
$$x_1, \quad x_2 \geq 0$$

(e) Prove that every Cauchy is bounded and converges to a real number.

(f) Show that $u = e^{-x}(x \sin y - y \cos y)$ is harmonic and find v such that $u + iv$ is analytic.

2. Answer the following five questions : 12×5=60

(a) Let G be the set of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where a, b, c, d are real numbers and $ad - bc \neq 0$. Show that G is an infinite non-Abelian group. 6+2+4

(b) If H be a subgroup of a group G and a, $b \in G$, then what do you mean by 'a is congruent to b mod H'? Show that the relation $a \equiv b (\text{mod}\, H)$ (a is congruent to b mod H) is an equivalence relation. 4+8

(c) What do you mean by 'an automorphism of a group G'? If $\mathscr{A}(G)$ be the set of all automorphisms of a group G, then show that $\mathscr{A}(G)$ is a group. 4+8

(d) Define ring, integral domain and field. Prove that any field is an integral domain. Is the converse true? Justify. 6+4+2

(e) Define a 'Boolean ring'. Prove that a Boolean ring is a commutative ring. Is the converse true? Justify. 2+6+4

3. Answer the following five questions : 12×5=60

(a) *(i)* What do you mean by 'a Riemann integrable function f on $[a, b]$'?

(ii) Show that every continuous function is R-integrable.

(iii) If f is R-integrable on $[a, b]$ and m and M be g.l.b. and l.u.b. of f on $[a, b]$, then show that for $b \geq a$

$$m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$$

2+5+5

(b) If $u = \tan^{-1} \dfrac{x^3 + y^3}{x - y}$, show that

(i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$

(ii) $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = 2\cos 3u \cdot \sin u$ 6+6

(c) Applying Cauchy's theorem and Cauchy's residue theorem, evaluate

$$\int_C \frac{z-3}{z^2 + 2z + 5} dz$$

where C is the contour

(i) $|z| = 1$

(ii) $|z + 1 + i| = 2$ 6+6

(d) Applying contour integration method, prove that

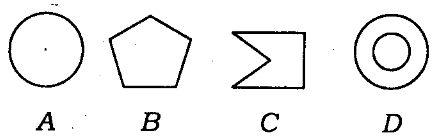
$$\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+1)(x^2+4)} = \frac{\pi}{3}$$ 12

(e) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $i, 0, -1$. Hence find the image of $|z| < 1$. 12

4. Answer the following four questions :

(a) (i) What do you mean by a convex region?

(ii) Which of the following regions are convex?



(iii) Examine whether union and intersection of convex regions are convex or not. 2+4+10

(b) Find by the graphical method the maximum value of $Z = 2x + 3y$, subject to the constraints

$$x + y \le 30, \quad y \ge 3$$
$$0 \le y \le 12, \quad x - y \ge 0$$
$$0 \le x \le 20$$ 16

(c) Define feasible solution, optimal solution, slack variables and surplus variables. 3×4=12

(d) Using simplex method, solve the following LPP : 16

$$\text{Max } Z = 5x_1 + 3x_2$$

subject to

$$x_1 + x_2 \le 2$$
$$5x_1 + 2x_2 \le 10$$
$$3x_1 + 8x_2 \le 12$$
$$x_1, \quad x_2 \ge 0$$

SECTION—B

5. Answer any *five* parts : 12×5=60

(a) Form the partial differential equations by eliminating the arbitrary functions from $Z = f(x+at) + g(x-at)$.

(b) Solve

$$\frac{\partial^2 z}{\partial x^2} + z = 0$$

given that when $x = 0$, $z = e^y$ and $\frac{\partial y}{\partial x} = 1$.

(c) Solve

$$(x^2 - y^2 - z^2)p + 2xyq = 2xz$$

(d) Apply Gauss elimination method to solve the equations $x + 4y - z = -5$, $x + y - 6z = -12$, $3x - y - z = 4$.

(e) Using Newton-Raphson method, solve the equations $x = x^2 + y^2$, $y = x^2 - y^2$ correct to two decimals, starting with the approximation $(0 \cdot 8,\ 0 \cdot 4)$.

(f) From the following table, estimate the number of students who obtained marks between 40 and 45 :

Marks	30–40	40–50	50–60	60–70	70–80
No. of Students	31	42	51	35	31

6. Answer the following questions : 12×5=60

(a) Following PDIs are associated with practical phenomena. Name the equations mentioning the associated phenomena : 3×4=12

(i) $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$

(ii) $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

(iii) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

(iv) $\begin{cases} -\dfrac{\partial V}{\partial x} = L\dfrac{\partial I}{\partial t} \\ -\dfrac{\partial I}{\partial x} = C\dfrac{\partial V}{\partial t} \end{cases}$

(b) Find a real root of the equation $x\log_{10} x = 1 \cdot 2$ by regula-falsi method correct to four decimal places. 12

(c) Evaluate

$$\int_0^6 \frac{dx}{1+x^2}$$

by using Simpson's $\frac{1}{3}$rd rule. 12

(d) Show that the moment of inertia of an elliptic area of mass M and semi-axes a and b about a diameter of length r is $\frac{1}{4}M\frac{a^2b^2}{r^2}$. 12

(e) (i) Where is the data for the 'hard disk type' stored in?

(ii) What is the capacity of DSDD floppy diskette?

(iii) Mouse is connected to which port?

(iv) Name a command which is not an internal DOS command. 12

7. Answer the following five questions : 12×5=60

(a) State D'Alembert's principle. Deduce the general equation of motion of a rigid body from D'Alembert's principle. 12

(b) What do you mean by holonomic system and non-holonomic system? Set up the Lagrangian for a simple pendulum, and obtain the equation describing its motion. 12

(c) (i) What do you mean by bit, byte and word?

(ii) Divide $1100_2 \div 10_2$.

(iii) Add hexadecimal numbers $6AE_{16} + 1FA_{16}$. 4+4+4

(d) Draw the truth table for the following : 4×3=12

(i) $Y = A.B + B.C$

(ii) $R = A(\overline{B} + \overline{C})$

(iii) 3-input OR-gate

(e) Create a sequential data file and store the serial number, name, basic pay, dearness allowance, house rent allowance, provident fund and LIC of 5 employees of a company. 12

8. Answer the following five questions : 12×5=60

(a) Use Lagrange's equations to find the differential equation for a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis. 12

(b) AB, BC are two equal similar rods freely hinged at B and lie in a straight line on a smooth table. The end A is struck by a blow perpendicular to AB; show that resulting velocity of A is $3\frac{1}{2}$ times of B. 12

(c) *(i)* Convert 38.21_{10} to its binary equivalent.

(ii) Convert 11011110101110_2 to hexadecimal.

(iii) Convert $B2F_{16}$ to octal. 4+4+4

(d) *(i)* What is programming?

(ii) Name the steps required for program development.

(iii) What is programming language? 4+4+4

(e) Write an algorithm to find whether a given number is odd or even. 12

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